TTK4150 Nonlinear Control Systems
Exercise 5
Part 2

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Exercise 1
In each of the following cases verify the describing function

1. *Let*
$$\psi(y) = y^5$$
then
$$\Psi(a) = \frac{5a^4}{8}$$

2. *Let*
$$\psi(y) = y^3 |y|$$
then
$$\Psi(a) = \frac{32a^3}{15\pi}$$

3. *Let the nonlinearity be given by Figure 1 then*
$$\Psi(a) = k + \frac{4A}{a\pi}$$

4. *Let the nonlinearity be given by Figure 2 then*
$$\Psi(a) = \begin{cases} 0 & \text{when } a \leq \Delta \\ \frac{4A}{a\pi}\sqrt{1 - \left(\frac{\Delta}{a}\right)^2} & \text{when } a \geq \Delta \end{cases}$$

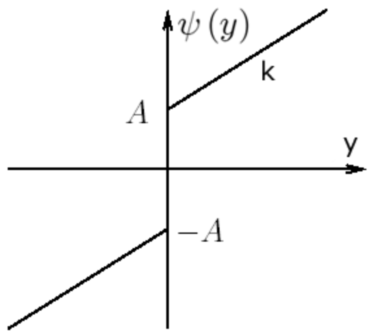

Figure 1: Nonlinearity

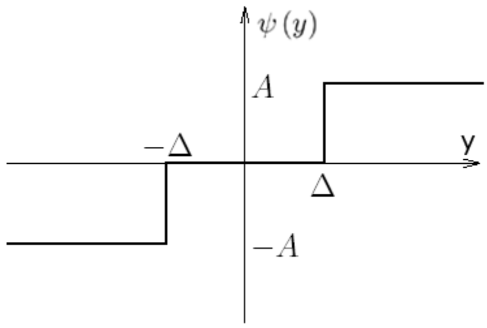

Figure 2: Nonlinearity

Exercise 2
Consider the system in Figure 3 where

$$\begin{aligned} h(s) &= \frac{1-s}{s(s+1)} \\ \psi(z) &= z^5 \end{aligned}$$

1. *Justify the use of the describing function method on this system.*
2. *Use analytic methods to investigate possible periodic solutions. If such a solution exist, estimate its frequency and amplitude.*
3. *Use graphical methods to investigate possible periodic solutions. If such a solution exist, estimate its frequency and amplitude.*
4. *If periodic solutions where found, determine their stability properties.*

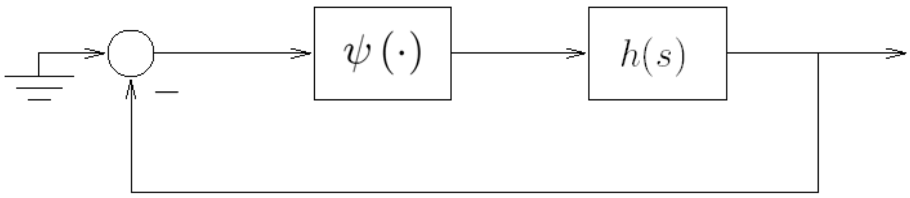

Figure 3: Closed loop system

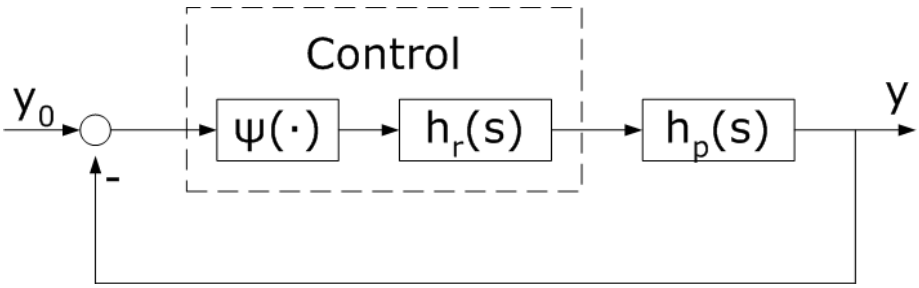

Figure 4: Thermostat control

Exercise 3

Consider the system in Figure 4 where the control law consists of a nonlinear function in series with some linear control law(the control law is given by $\psi(\cdot)$ and $h_r(s)$). This can represent a thermostat control law where the process typically describes temperature dynamics of a room. Let

$$h_r(s) = \frac{K}{s},\ K > 0$$
$$h_p(s) = \frac{1}{1+Ts},\ T > 0$$

and $\psi(\cdot)$ be a relay with hysteresis as given in Figure 1.12 in Khalil. Further, let y be the Celsius temperature of the room and $y_0 = 0$ be the desired temperature (the equilibrium is shifted to a desired equilibrium).

1. *Justify the use of the describing function method on this system.*
2. *Derive the describing function of the nonlinearity $\psi(\cdot)$.*
3. *Plot $-\frac{1}{\Psi(a)}$ in a Nichols diagram as a function of $\frac{a}{S}$ (the magnitude is then scaled by $\frac{S}{L}$).*

4. *Let*

$$\begin{aligned} K &= 20 \\ T &= 1 \\ L &= 1 \\ \Delta &= 1 \end{aligned}$$

Use a graphical method to estimate the amplitude and frequency of the periodic solutions in the system.

5. *Are the periodic solutions stable? What does these periodic solutions tell you about the temperature in the room?*

6. *Simulate the system and comment with respect to the results found when using the describing function method. Try different initial conditions.*

7. *What can be done to reduce the amplitude of the oscillations.*

A Describing functions

Let $\Psi(a,\omega)$ be the describing function of the nonlinearity $\psi(\cdot)$. Further, let

$$\begin{aligned} z(t) &= \psi(y(t)) \\ y(t) &= a\sin(\theta) \\ \theta &= \omega t \end{aligned}$$

then an approximation of $z(t)$ is given by

$$z(t) \approx z_0 + z_1 \sin(\theta + \varphi)$$

where only the first order Fourier coefficients have been used in the approximation. The various parameters are given by

- $z_0 = \frac{1}{2\pi}\int_0^{2\pi} \psi(a\sin(\theta))\, d\theta$
- $z_1 = \sqrt{z_{1s}^2 + z_{1c}^2}$
  - $z_{1s} = \frac{1}{\pi}\int_0^{2\pi} \psi(a\sin(\theta))\sin(\theta)\, d\theta$
  - $z_{1c} = \frac{1}{\pi}\int_0^{2\pi} \psi(a\sin(\theta))\cos(\theta)\, d\theta$
- $\varphi = \arctan\left(\frac{z_{1c}}{z_{1s}}\right)$

In the case of odd nonlinearity $\psi(\cdot)$, the describing function is given by

$$\begin{aligned} |\Psi(a,\omega)| &= \frac{z_1}{a} \\ \angle\Psi(a,\omega) &= \varphi \end{aligned}$$