TTK4150 Nonlinear Control Systems
Exercise 6
Part 2

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Exercise 1

In the following we are going to develop a control law for a spacecraft based on passivity. The model of the spacecraft is described with two state vectors. The orientation dynamics is given by

$$\dot{\phi} = R(\phi)\,\omega$$

and the Euler equations for a rotating rigid spacecraft are given by

$$\begin{aligned} J_1\dot{\omega}_1 &= (J_2 - J_3)\,\omega_2\omega_3 + \tau_1 \\ J_2\dot{\omega}_2 &= (J_3 - J_1)\,\omega_3\omega_1 + \tau_2 \\ J_3\dot{\omega}_3 &= (J_1 - J_2)\,\omega_1\omega_2 + \tau_3 \end{aligned}$$

where $\phi = [\phi_1\ \phi_2\ \phi_3]^T$ *is the orientation vector,* $\omega = [\omega_1\ \omega_2\ \omega_3]^T$ *is a vector of angular velocity along the principal axes,* $\tau = [\tau_1\ \tau_2\ \tau_3]^T$ *is a vector of torque inputs applied about the principal axes and* J_i *are the principal moments of inertia. Further, we have that* $R(\phi) = R^T(\phi)$ *and we assume that* $R(\phi)$ *is non singular in our domain of interest (*$R(\phi)$ *is non singular on* D_ϕ *but not on* $\mathbb{R}^3$*).*

1. *Before we start with the passivity analysis and control design, the model properties are investigated and located.*

   (a) *Verify that the system may be rewritten in a compact form as*

$$\begin{aligned} \dot{\phi} &= R(\phi)\,\omega \\ M\dot{\omega} &= C(\omega)\,\omega + \tau \end{aligned}$$

where M is positive definite and symmetric and $C(\omega)$ skew symmetric.

(b) *We are solving the feedback stabilization problem for the spacecraft. Our stabilization problem is defined as stabilizing the system with a desired orientation and zero angular velocity. In order to apply our theoretical tools we need to rewrite the system such that origin is the point of interest. Let $e_1 = 0$ and $e_2 = 0$ describe the point of interest of ϕ and ω respectively, and rewrite the system in these variables.*

2. *Passivity based control consists mainly of two steps; locating a suitable input-output pair based on passivity and force some stronger passivity properties to the input-output pair, if not already present. We will conduct our passivity analysis using the storage function*

$$V(e) = \frac{1}{2}e_2^T M e_2 + \frac{1}{2}e_1^T K_p e_1$$

where $K_p = K_p^T > 0$. In our case the control input is given by τ, and we only need to find a suitable measurement (output).

   (a) *What physical interpretation can be made of the storage function.*

   (b) *Find a suitable output based on passivity analysis. Justify your answer.*

   (c) *Does the system have any passivity properties with respect to your choice of output and τ.*

3. *The next step, after having chosen a suitable output, is to force some passivity property on the system using the control input.*

   (a) *Find a control input $\tau(\phi, \omega, v)$ such that the system is passive (lossless) from v to your choice of output.*

   (b) *Comment on stability properties achieved with this control input.*

   (c) *Find a control input $\tau(\phi, \omega, v)$ such that the system is output strictly passive from v to your choice of output.*

   (d) *What are the differences of the control input turning the system output strictly passive with respect to the control input turning the system passive? Give a physical interpretation.*

   (e) *Comment on stability properties achieved with this control input.*

4. *Suppose that we desire a more complex control law for the overall system. We desire to add control without making any calculations.*

   (a) *Which of the previous control laws would you base your extension on? Justify the choice.*

   (b) *The initial control law $\tau(\phi, \omega, 0)$ is troubled with a steady state deviation in the rotational velocity. Propose a control law that is likely to solve this problem. Is it possible to use the same approach to cope with a steady state deviation of the orientation.*

   (c) *What can be said of the robustness with respect to uncertainty in measurements and actuators of the proposed system.*

Exercise 2 (Exercise 14.30 a) in Khalil)

Exercise 3 (Exercise 14.31 in Khalil)

Exercise 4 (Exercise 14.34 in Khalil)