Take Home Exam

TTK 4190 Guidance and Control

Due date: 2nd March 2004 10:00 AM

Remember to write your NTNU student number on your solution

Instructions. You may submit your solution in Norwegian or English. The exam is due March 2nd at 10:00 AM. By then, you have to post your solutions in a box marked "*Hand-inn box for TTK4190 Guidance and Control*" located in room D238 at the Department of Engineering Cybernetics. If you are not capable of delivering a paper version of your solution (e.g. persons not in Trondheim at the moment), you may submit your solution electronically to the teaching assistant (TA). Please contact the TA as soon as possible to make arrangements for electronic submission.

1 Theory

1.1 Rotation matrix

Verify that the rotation matrix corresponding to the unit quaternion is given by

$$\mathbf{R}_b^n(\mathbf{\Theta}) = \begin{bmatrix} 1-2(\epsilon_2^2+\epsilon_3^2) & 2(\epsilon_1\epsilon_2-\epsilon_3\eta) & 2(\epsilon_1\epsilon_3+\epsilon_2\eta) \\ 2(\epsilon_1\epsilon_2+\epsilon_3\eta) & 1-2(\epsilon_1^2+\epsilon_3^2) & 2(\epsilon_2\epsilon_3-\epsilon_1\eta) \\ 2(\epsilon_1\epsilon_3-\epsilon_2\eta) & 2(\epsilon_2\epsilon_3+\epsilon_1\eta) & 1-2(\epsilon_1^2+\epsilon_2^2) \end{bmatrix}$$

1.2 Invariance of the unit quaternion with respect to the rotation matrix

Prove that the unit quaternion is invariant with respect to the rotation matrix and its transpose, i.e.

$$\mathbf{R}(\eta,\epsilon)\epsilon = \mathbf{R}^T(\eta,\epsilon)\epsilon = \epsilon$$

1.3 Elementary rotations

Consider the elementary rotation about coordinate axes given by infinitesimal angles. Show that the rotation resulting from any two elementary rotations does not depend on the order of rotations.

Further define $\mathbf{R}(d\phi, d\theta, d\psi) = \mathbf{R}(d\phi)\mathbf{R}(d\theta)\mathbf{R}(d\psi)$, and show that :

$$\mathbf{R}(d\phi, d\theta, d\psi)\mathbf{R}(d\phi', d\theta', d\psi') = \mathbf{R}(d\phi + d\phi', d\theta + d\theta', d\psi + d\psi')$$

1.4 Rotation matrix property

Prove the equation:

$$\mathbf{R}\mathbf{S}(\omega)\mathbf{R}^T = \mathbf{S}(\mathbf{R}\omega)$$

1.5 Quaternion propagation

Prove the so called *quaternion propagation* equation:

$$\begin{aligned}\dot{\eta} &= -\tfrac{1}{2}\epsilon^T\omega \\ \dot{\epsilon} &= \tfrac{1}{2}(\eta I - S(\epsilon))\omega\end{aligned}$$

In solving this it might be useful to consider the following notes.

Note 1. $\omega = 2S(\epsilon)\dot{\epsilon} + 2\eta\dot{\epsilon} - 2\dot{\eta}\epsilon$

Note 2. $\dot{\epsilon}^T\epsilon = \epsilon^T\dot{\epsilon} = -\eta\dot{\eta}$

Figure 1: Supply vessel.

2 Computer Simulations

Consider a 3 DOF supply ship model in the form:

$$\dot{\psi} = r \tag{1}$$

$$\underbrace{\begin{bmatrix} m_{11} & 0 & 0 \\ 0 & m_{22} & m_{23} \\ 0 & m_{32} & m_{33} \end{bmatrix}}_{\mathbf{M}} \begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{r} \end{bmatrix} + \underbrace{\begin{bmatrix} d_{11} & 0 & 0 \\ 0 & d_{22} & d_{23} \\ 0 & d_{32} & d_{33} \end{bmatrix}}_{\mathbf{D}} \begin{bmatrix} u - u_c^b \\ v - v_c^b \\ r \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix} \tag{2}$$

where u is the surge velocity, v is the sway velocity, r is the yaw rate, ψ is the yaw angle, (u_c^b, v_c^b) are the longitudinal and transverse current velocities given by:

$$
\begin{aligned}
u_c^b &= V_c \cos(\beta_c - \psi) & (3)\\
v_c^b &= V_c \sin(\beta_c - \psi) & (4)\\
\dot{V}_c &= 0 & (5)\\
\dot{\beta}_c &= 0 & (6)
\end{aligned}
$$

The numerical values are:

```
M =...
  [6.7644e6         0         0
          0  1.1341e7 -5.8583e3
          0 -5.8583e3  1.3206e2]
D =...
  [7.7032e4         0         0
          0  2.5455e5 -3.5015e2
          0 -1.1578e2  1.1414e1]
```

while the current is given by:

$$
\begin{aligned}
V_c &= 0.5 \text{ m/s} & (7)\\
\beta_c &= 45.0 \text{ deg} & (8)
\end{aligned}
$$

2.1 Hydrodynamics

Write down all model parameters m_{ij} and d_{ij} as a function of the hydrodynamic coefficients and rigid-body parameters, that is:

$$
\begin{aligned}
m_{11} &= m - X_{\dot{u}} & (9)\\
&\quad \text{etc.} & (10)
\end{aligned}
$$

and explain the physical meaning of: $Y_{\dot{v}}, Y_{\dot{r}}, N_{\dot{r}}, I_z, Y_v$ and N_v.

2.2 State space modeling and observability

The ship-current model is nonlinear in ψ but a linear state space model can be found by assuming that:

$$
\begin{aligned}
\dot{u}_c^b &= 0 & (11)\\
\dot{v}_c^b &= 0 & (12)
\end{aligned}
$$

such that:

$$\mathbf{x} = [\psi, u, v, r, u_c^b, v_c^b]^\top \tag{13}$$

Write down the $\mathbf{A}$ and $\mathbf{B}$ matrices in the state-space model:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\boldsymbol{\tau} \tag{14}$$

and explain for which maneuvers this model can be used, that is discuss the validity of the assumption.

2.3 Observability

Let the measurement equation be denoted as:

$$\mathbf{z} = \mathbf{H}\mathbf{x} + \mathbf{v} \tag{15}$$

1) Consider the linear state-space model and assume that the only measurement is yaw angle:

$$z = \psi + \text{white noise} \tag{16}$$

Write down the **H** matrix. Is this model observable?
2) Assume that you have three measurements (yaw angle ψ and velocities u and v):

$$\mathbf{z} = \begin{bmatrix} \psi \\ u \\ v \end{bmatrix} + \text{white noise} \tag{17}$$

Write down the **H** matrix. Is this model observable?

2.4 Kalman filtering

Write down the continuous-time steady-state Kalman filter equations for the linear ship-current model using the three measurements (ψ, u, v) and simulate the Kalman filter in Matlab for different inputs:

$$\boldsymbol{\tau} = \begin{bmatrix} A_1 \sin(\omega_1 t) \\ A_2 \sin(\omega_2 t) \\ A_3 \sin(\omega_3 t) \end{bmatrix} \tag{18}$$

where A_i and ω_i $(i = 1, 2, 3)$ are design parameters.

Include plots from the following variables in your report:

- control inputs $\tau_i = A_i \sin(\omega_i t)$ as a function of time t
- corresponding estimates $(\hat{u}, \hat{v}, \hat{r}, \hat{\psi})$ and $(\hat{u}_c^b, \hat{v}_c^b)$ as a function of time t

All estimates should converge to their true values, that is $\hat{\mathbf{x}} \to \mathbf{x}$. Explain why this is possible?

2.5 Nonlinear Kalman filtering

In the general case $u_c^b = V_c \cos(\beta_c - \psi)$ and $v_c^b = V_c \sin(\beta_c - \psi)$ introduce a nonlinearity in the state-space model. Show how a Kalman filter can be designed for the nonlinear state-space model. No simulations are required, just the basic equations of the filter.

2.6 Nomoto models

Compute the gain and time constants in the <u>two</u> Nomoto models:

$$\frac{\psi}{\tau_3}(s) = \frac{K(1 + T_3 s)}{s(1 + T_1 s)(1 + T_2 s)} \approx \frac{K}{s(1 + Ts)} \tag{19}$$

and plot the transfer functions in one Bode plot using Matlab. What is the supply vessel cross-over frequency in yaw?

System disturbance matrix

$$E = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Filter design matrices

$$R = 10 \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad Q = 10 \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Inital conditions

$$P = 100 \cdot \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1000 & 0 \\ 0 & 0 & 0 & 0 & 0 & 100 \end{bmatrix}, \quad x_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 2 \\ 2 \end{bmatrix}, \quad \hat{x}_0 = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \\ 10 \\ 1 \end{bmatrix}$$

Noise parameters

$$\mathbf{v} = \mathbf{w} = white\ noise \begin{bmatrix} mean(=0), & var(=0.1) \\ mean(=0), & var(=0.1) \\ mean(=0), & var(=0.1) \end{bmatrix}$$

Table 1: Parameters for simulating the Kalman filter

2.7 Heading autopilot design

We will consider transit operations offshore. For this purpose a heading autopilot will be designed. Assume that:

$$\begin{aligned} \tau_1 &= 1.5406 \cdot 10^5 \quad \text{(N)} && (20) \\ \tau_2 &= 0 \text{ (N)} && (21) \\ \tau_3 &= \text{control input used for turning} && (22) \end{aligned}$$

The problem is to design τ_3 such that the ship can be turned during transit (τ_1 = constant). This is usually done by designing a PID-controller for heading control. Compute K_p, K_d and K_i using the Nomoto constants (K, T) and your specifications for natural frequency ω_n and relative damping ratio ζ. Simulate the PID-controller in closed loop for the following two cases:

- nonlinear ship-current model (no state estimator)
- nonlinear ship-current model with feedback from estimated states

Include plots of the following variables in your report:

- reference input ψ_d changing from 0^o to 40^o as a function of time t. The reference signal should be low-pass filtered.
- corresponding states $(u, v, r, \psi, u_c^b, v_c^b)$ and estimates $(\hat{u}, \hat{v}, \hat{r}, \hat{\psi}, \hat{u}_c^b, \hat{v}_c^b)$ as a function of time t

2.8 Integral action

Explain why we need integral action in the controller and observer and make comments to your simulation results.

2.9 Feedback from $\dot{r}$

What can be achieved by including feedback from $\dot{r}$? Is it possible to measure $\dot{r}$ using a conventional sensor?